

# STEEPEST GRADIENT METHOD

Direct differential gradient method  
Climbing hill method

## 1. Example without constraint

Objective function:  $ZQ = CX2 * X^2 + CY2 * Y^2 + CX * X + CY * Y + CK$   
Constraint area: Nothing

### Example 1

Objective function:  $Z = 9 * X^2 + 4 * Y^2 - 72 * X - 64 * Y$

| Objective function: ZQ |     |     |     |     |    |
|------------------------|-----|-----|-----|-----|----|
|                        | CX2 | CY2 | CX  | CY  | CK |
| Example 1              | 9   | 4   | -72 | -64 | 0  |

Make ZQ , Max, or, Min

Function of 1 step:

$$q = S * p$$

, where,  $p = \text{grad}(ZQ)$ , that is,  
 $= [\delta z / \delta x, \delta z / \delta y]$

$S = \text{constant}$ : Example 1: 0.001

Start point, X, Y: 0, 0

Epsilon for stop: EP = 0.00001,

Repeat times for stop: NN= 10000

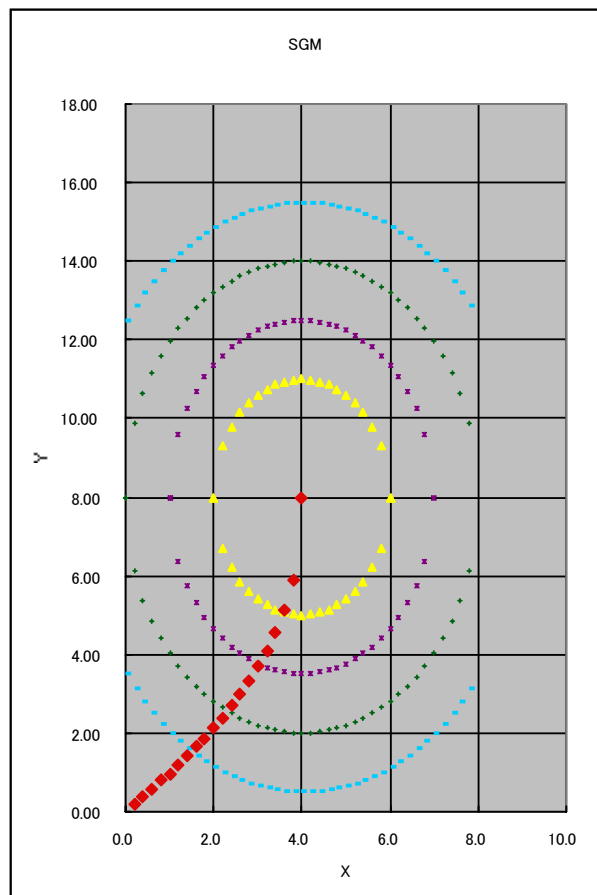
### Answer 1

$$ZQ = 9 * (X-4)^2 + 4*(Y-8)^2 - 400$$

$$\delta z / \delta x = 18 * (X-4)$$

$$\delta z / \delta y = 8 * (Y-8)$$

$$ZQ \text{ min} = -400$$



Final value(Optimal value)

Repeat times J =

| X     | Y    | Z      |
|-------|------|--------|
| 4.00  | 8.00 | 400.00 |
| 10001 |      |        |

See [sgm-01.xls](#): [sgm-1-a]

## 2. Example with constraint

Objective function:  $ZQ = CX2 * X^2 + CY2 * Y^2 + CX * X + CY * Y + CK$

Constraint area:  $G = G(X, Y) = KX * X + KY * Y + KC \leq 0$

### Example 2

Objective function:  $Z = 9 * X^2 + 4 * Y^2 - 72 * X - 64 * Y$

Constraint area:  $G = 2 * X + Y - 8 \leq 0$

|                        |     |     |     |     |    |
|------------------------|-----|-----|-----|-----|----|
| Objective function: ZQ |     |     |     |     |    |
|                        | CX2 | CY2 | CX  | CY  | CK |
| Example 2              | 9   | 4   | -72 | -64 | 0  |

Constraint area: G

|           |    |    |    |
|-----------|----|----|----|
|           | KX | KY | KC |
| Example 2 | 2  | 1  | -8 |

Make ZQ, Max, or, Min

Function of 1 step:

$$q = S * p - u * r$$

where,  $p = \text{grad}(ZQ)$ , that is,

$$= [\delta ZQ / \delta x, \delta ZQ / \delta y]$$

$r = \text{grad}(G)$ , that is,

$$= [\delta G / \delta x, \delta G / \delta y]$$

$u: u = 0$  if  $G \leq 0$ ,  $u = G(X,Y)$  if  $G > 0$

$S = \text{constant}$ :

$$S = 0.001 \text{ if } G \leq 0,$$

$$S = 0 \text{ if } G > 0$$

### Answer 2

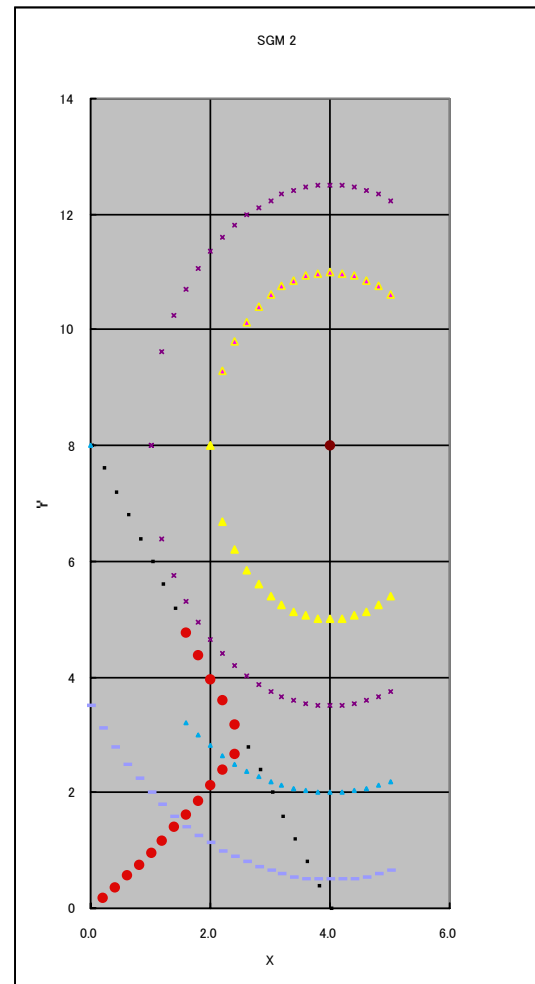
$$ZQ = 9 * (X-4)^2 + 4 * (Y-8)^2 - 400$$

$$\delta z / \delta x = 18 * (X-4)$$

$$\delta z / \delta y = 8 * (Y-8)$$

$$\delta G / \delta x = 2$$

$$\delta G / \delta y = 1$$



Final value(Optimal value)

Repeat times J =

| X     | Y    | Z      |
|-------|------|--------|
| 1.44  | 5.12 | 307.84 |
| 29395 |      |        |

See [sgm-01.xls](#): [sgm-2-a] 2004/8/18